Plenary tutorial 0

- Before I even begin, just recall some dimensional analysis,

By setting $[\hbar] = [c] = 1$, we get

(i) $1 = [c] = LT^{-1} \Rightarrow [T] = [L]$

(ii) $1 = [\hbar] = ML^2T^{-1}$

$\Rightarrow 1 = ML \Rightarrow [M] = [L]^{-1}$

Combining these we get, $\boxed{[M] = [T]^{-1} = [L]^{-1}}$

- Just from $E = mc^2 \Rightarrow E = m \Rightarrow \therefore [M] = \underline{\underline{eV}}$

$[T] = eV^{-1}$

$[L] = eV^{-1}$

- Notes if you pay attention while taking them (not me)
- No notes if you get distracted!

(No need to take notes - Just lay back and hear me blab)

1. Scales in QFT (What does it mean when we say "High energy physics") From 4-gravitons blog, just a very cool blog.

So to look at v.short distances (or take them apart), we need v.large momenta & hence v.large E!

$\Delta x \Delta p \geq \frac{\hbar}{2}$

- **'High' vs 'Low' classifies what kind of Physics you are dealing with**
  - **'High energy physics' also corresponds to 'very small distance scale physics'**
  - **'Low energy physics' also encompasses to larger structures**
- **Why and how? Heuristically -**
  - **From QM :** HUP tells you that the more certain you are of a particle's position, the more and more uncertain you are about it's momentum and hence energy, i.e. it can have large energies.
  - **Classically :** If you hold two things very close together - at extremely small scales, it generally takes a lot of force to keep them from flying away, so such bonds will have more energy
- **Terminology**
  - IR means low energy - IR has longer wavelength, so less energy
  - UV means high energy - UV has shorter wavelength so more energy

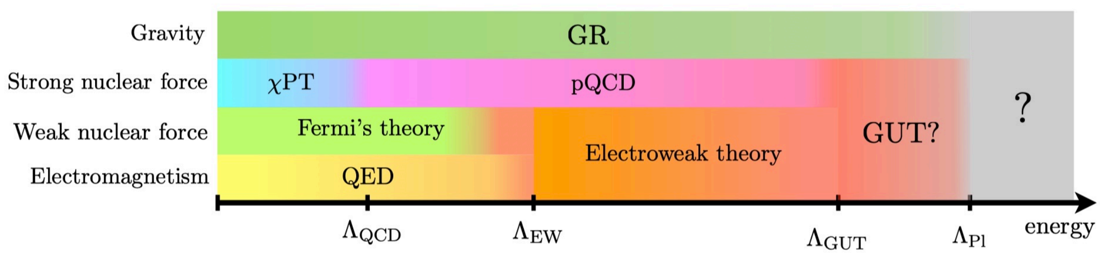


Fig. 1: Simplified cartoon showing the network of EFTs behind our understanding of subatomic physics.

We can have few limits from lowest energies to highest :

1. **IR limit :** Lowest of "low energy", limit of infinitely low energy. One may compare it to "absolute zero", it means very little energy compared to other energies in your calculation/system, that you might as well take it zero
2. **Low energy physics :** From absolute zero to atoms, Once you have enough energy to break up atoms, you get,
3. **High energy physics :**
   A. Also known as 'particle physics'. Subatomic particles (Not strings or Branes)
   B. If you leave nuclear physics out, then its MeV and higher energies (Neutrinos are many times taken into low energy physics cause they don't have enough energy to break up atoms and are quite feeble - again this is just nomenclature, nothing set in stone)
   C. So in terms of chemistry, electrons in atoms are at eV energies, so HEP regime is atleast a million times more energetic than the characteristic energy of chemistry. (Again, I am generalizing as a typical theoretical physicist, surprise surprise, just be glad I am not giving energies of spherical chickens)
4. **TeV scale :**
   A. This is a bit special, as if you are working in collider physics, MeV is these days considered low energy scales as we have achieved energies of TeV at the Tevatron lets say.
   B. Quarks which are MeV scale are often taken as massless in when you are dealing with GeV, TeV energies.
5. **String / GUT scale:**
   A. Strings : Flexible but with enormous tension that keep them EXTREMELY short.
   B. How short? Planck scale, so this is the scale where Gravity effects become important

C. The corresponding energy of Planck scale : 10^28 eV, this is absolute bonkers amount of energy!
   a. Atleast about 10000000000 times more energy than what I called "High energy or TeV"
D. When people talk about curled up dimensions, extra dimensions, its at these scales. The main issue is, these all will stay theories till we reach those energies, or find a way to probe so early in the universe where these energies according to our standard model of cosmology were common (This is actually an active field, superstring cosmology and primordial gravitational waves**)**

6. **UV Limit:** Formal limit of infinitely high energy.

(Remember, IR and UV light are not accurate in terms of these scales, both are around meV to eV scale, its just NOMENCLATURE)

- **Let us put everything that we did into examples, without even getting into too much math or details, we will just keep it physically relevant right now.**

EXAMPLE 1

$$S = \int \underbrace{d^4x}_{E^{-4}} \left(\frac{1}{2} \underbrace{\partial_\mu \phi}_{E} \underbrace{\partial^\mu \phi}_{E} - \frac{m^2}{2}\phi^2 - \frac{\overset{?}{\lambda_4}}{4!}\phi^4 - \frac{\overset{?}{\lambda_6}}{6!}\phi^6 \right).$$

ϕ: E each

This is the action, has units of $\hbar$, so dimless in NU. (Called 'Quantum of action')

$\Rightarrow$ $[\lambda_4]$ = Dimensionless
$[\lambda_6]$ = E^{-2} ! $\rightarrow$ Terms which have negative coupling constants are called 'Higher dimensional operators'. (HDO)
$\rightarrow$ So ϕ^6 is the HDO.

Q. What does having these higher dimensional operators imply?
↳ We are dealing w/ a theory that is NOT FUNDAMENTAL.

Q. Ok, what do you mean by 'NOT FUNDAMENTAL'?
- It is only valid at energies $\boxed{E \ll \Lambda}$ & as $E \to \Lambda$, it should be replaced by some more fundamental theories. i.e. as Energies $E \to \Lambda$, this theory starts making less & less sense.
- COMMENT : We could also have $\underbrace{E_1 < \Lambda_1}_{\text{Theory 1}} < \underbrace{E_2 < \Lambda_2}_{\text{Theory 2}}$.
  ↳ We will see an example of this in QCD (maybe in lectures?, I'll brief today what this means).

- Technically, we can put ∞ such HDO in our Lagrangian
  ↳ We can club all these into (for $n>4$)

$$S \supset \frac{g_n}{\Lambda^{n-4}} \int d^4x \, \mathcal{O}_n$$

g_n: Dimensionless

$\rightarrow [\mathcal{O}_n] = E^{n-4}$

- So the correction to this part of S scales as $(E/\Lambda)^{n-4}$, & recall $\Lambda \gg E$, so there is a clear heirarchy (of which term or HDO contributes how much set by this)

- This is the basics of $\boxed{\text{EFT}}$.
  ↳ Due to our experimental limitations, (Going to higher energies, Detector Sensitivity) (say we want to look at BSM physics), we just need to include a few HDO's while computing physical variables. (Or even SM physics btw)

[Have you seen the spring-ball description of QFT?? If yes, then the scale there is $\Lambda \sim \frac{\hbar c}{a}$ gives $\hbar\omega_{max}$.
↳ Going lower in a, gives higher $\hbar\omega_{max}$, HIGH ENERGY]

- Lets take another example (In (1+3) dimensions)

$\phi\,(\partial_\mu\phi)$ Derivative Couplings are interesting!

$$S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}c_m \Lambda^2 \phi^2 - c_3 \Lambda \phi^3 - c_4 \phi^4 \underbrace{- \frac{c_5}{\Lambda}\phi^5 - \cdots - \frac{g_3}{\Lambda}\phi(\partial_\mu\phi) - \frac{g_4}{\Lambda^2}\phi^2(\partial_\mu\phi) - \cdots}_{\text{Suppressed by } \left(\frac{1}{\Lambda}\right)^n} \right)$$

Can get momentum dependence in your amplitudes/observables

↳ If we are conducting an experiment at $E \ll \Lambda$, ($R \gg a$) all but the first 4 terms are heavily suppressed!
(At $E \sim \Lambda$ those terms will start getting relevant)
Why only one Λ here? Single scalar field assuming its valid at $E \ll \Lambda$.

↳ At low energies (long-distances), we can ignore all of that & then the theory of single scalar field in a L.I. universe is given by,

$$S_{low-energy} \approx \int d^4x \left(\underbrace{\frac{1}{2}(\partial_\mu \phi)^2}_{\text{Kinetic}} - \underbrace{\frac{1}{2}c_m \Lambda^2 \phi^2}_{\text{Mass}} - \underbrace{c_3 \Lambda \phi^3 - c_4 \phi^4}_{\text{Interactions}} \right)$$

- This is actually the Higgs field (RIP to the giant!) without any interactions.

Reference to Newton's quote, I am not size-shaming anyone...

- Due to symmetry, $\phi \to -\phi$, $c_3 = 0$.

- The HIERARCHY PROBLEM is that if Λ is really big (say Planck scale $\sim 10^{19}$ GeV), we must have $C_m \lll 1$! Very fine tuned such that $m_H \sim 125$ GeV !

Side note:
- RN & RG & loops things like that really are the essence of QFT, a lot of QFT results up to a good order can be obtained from QM, you really need QFT & loops to get that 15^{th} decimal from your experiment to match the theory.

- Why did I randomly bring planck scale? Let's see the relevance of this scale

EXAMPLE 2.

- Consider fluctuations around Minkowski metric

$$S = \frac{1}{16\pi G_N}\int d^4x \sqrt{-g}R$$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$$

$$\kappa = \sqrt{8\pi G_N}$$

Expand in powers of $h_{\mu\nu}$ & you'll get an interacting theory of gravitons (propagating) in flat spacetime)

- The interaction part looks like: $S_{\text{int}} = \sum_{n=3}^{\infty} \kappa^{n-2}\int d^4x\, \underbrace{\mathcal{O}_{n+2}[h,\partial]}_{\text{Dim}\,=\,n+2}$

At leading order, n=3, we have κ^1, which has dimensions of E^{-1}.

$$\boxed{[\kappa] = E^{-1}}$$

- In the spirit of EFT, this means that Einstein's gravity - not fundamental - but effective theory at energies valid below its **natural energy scale** set by the dimensional gravitational constant "Planck scale":

$$\Lambda_{\text{Pl}} \equiv \sqrt{\frac{\hbar c^5}{8\pi G_N}} = 2.4 \times 10^{18}\ \text{GeV},$$

$\sim 10^{14}$ larger than COM energies at LHC.

- GR cannot be quantized is not entirely a correct statement, the above action can be consistently quantized as long as we operate at $E << \Lambda_{Pl}$
    - In this regime Quantum fluctuations of the background metric as suppressed as E/Λ_{Pl}

- In other words, below the Planck energy scale quantum gravity is just a theory of weakly coupled gravitons propagating on a regular background spacetime.

- What about when we start approaching the Planck scale?
    - NOT an expert, but from what I know - They become strongly coupled and they cease to be the correct degrees of freedom for Quantum Gravity.

- What is the right statement to make about QG then?
    - Experimentally we have nothing - but we do have a consistent theory of quantum gravity low energies, but lack one which remains valid at arbitrarily high energies.

- The difference is crucial - latter kind of theory needed to analyze - what happens close to spacetime singularities, where quantum effects are so large as to override the semiclassical description provided by GR

- Viewed as an EFT, Einstein's (quantum) gravity is expected to be subsumed near Λ Pl into another theory, its ultraviolet (UV) completion, which presumably remains valid to arbitrarily high energies - like string theory.

RN and UV divergences (Quick note)

- All QFTs that describe SM physics, give us infinities when computing quantum corrections of (hbar or higher) to the original classical results
  - The origin of these infinites is - Quantum fluctuations of very small wavelength (high energies) dominate the result ending in Divergences
- This was addressed in 1940s, goes by the name Renormalization! (This is not really cutting off infinities but much much more important)

Three steps of RN

- Regularize : Put a cutoff, so fluctuations of wavelength Lambda^-1 are ignored
  - This makes results finite, but artificially as all physical observables depend on the cutoff scale you choose Lambda

- The main observation is that now, the physical parameters (masses, couplings, etc) in action need to be 'rescaled' appropriately (I am not sure if rescaled is the perfect word, but we will go along with it)
  - give a quick example of the electron bare mass if time permits else move on - will be addressed in the next tutorial)

- Put together 1+2 (along with counterterms which come as a part of the package), we get rid of this artificial cutoff dependent results. (Understand carefully, we are not saying that the physics is the same at all energies, we are saying if we are operating the theory at 1 eV and the cutoff is 100 GeV , then even if I choose the cutoff to be 90 GeV or 110GeV, we should get results that are the same at that low energies.

While going through this procedure there are two things that can happen

- (1) Divergences are removed with a finite number of operators 'counterterms' - most of the times, one for each present in the classical theory
  - These are Renormalizable theories - Example : SM

- (2) Infinite counterterms are needed to get rid of all the divergences,
  - Non-renormalizable theories - Example : GR

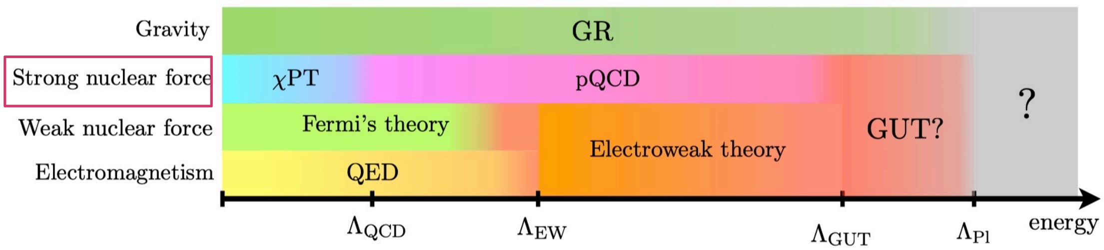
(Quick rule of thumb) : - Theories with operators having dimension <= 4 are RN (in 3+1 dimensions).
- Presence of higher operators are indicators of Non RN theories when working in Perturbation theory

- For a long time Renormalizbility of a theory was crucial for a theory to be 'physical' (EW is a very good exmaple where it was only taken seriously when it was mathematically proven to be RNable)

- MODERN PERSPECTIVE?

  - Needing to be RNable is too restrictive, equivalent for it to be valid at all all energies.
  - We don't exclude non-RNable theories off the bat.
  - They are EFTs whose natural energy scale is set by cutoff Lambda, they give accurate results for E<<Lambda
  - From this perspective, the cutoff gets a PHYSICAL RELEVANCE - its not a just a mathematical artifact anymore.
    - What is its Physical significance?
      - It is the energy threshold of the unknown physics encoded in the HDO of our EFTs
- Summary? NonRNability is basically a hint that there is some unknown physics hiding at high energies

• DEMONSTRATE ↑ w/ QCD : Interaction of Quarks & Gluons.



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• QCD : RN theory that can be modeled well upto arbitrarily high energies

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Asymptotically free : Meaning, at high energies, the coupling constant goes to ZERO.

This means, at low energies, we get very strongly coupled.
(Similar to how we said that what happens to Quantum GR at E_{planck})

• Because they become so strongly coupled, $\alpha \sim O(1)$, perturbative methods need to be abandoned (pQCD)

• Contrast to GR where we need to go at arbitrarily high energies to make sense at the non-perturbative scenario, life is simpler in QCD,

Why? We need to look at low energy regime i.e. do experiments!

↳ We get Baryons (qqq) & mesons ($q\bar{q}$) at these scales

↳ These are composite states whose interactions are heavily limited by symm.

• The framework is called χPT (Chiral perturbation theory), it's a nonRN theory, lot of high dim operators.

V.IMP : The cutoff Λ is not a random energy but, a physical scale $\boxed{\Lambda_{QCD}}$ below which Quarks are confined into hadrons !
(Baryon + Meson)

- Hadron interaction is an EFT valid at $E \ll \Lambda_{QCD}$.