Color Superconductivity in Dense Quark Matter

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Abstract

The behavior of quarks under extreme conditions has been a subject of intense research in recent decades. At high densities, quarks can form a novel state of matter known as quark matter, which has unique properties that have garnered attention from both particle physicists and astrophysicists. In this essay, we explore the phenomenon of color superconductivity, which is a manifestation of the behavior of quarks in dense quark matter. We discuss the various phases of quark matter, including the superconducting quark matter phase, the CFL phase, and the 2SC phase, as well as the dependence of the QCD phase diagram on the strange quark mass. Furthermore, we investigate how color superconductivity manifests in compact stars, discussing the challenges of maintaining beta equilibrium and charge neutrality and how these conditions can disfavor traditional color superconductors. However, we also explore how other types of superconductors can be formed due to a fermi momentum mismatch.

1 Dense Quark Matter

The concept of quark matter, a unique state of matter that can emerge at extremely high densities, is a topic of significant interest in the field of physics. However, it may be difficult to imagine encountering such conditions in our daily lives. To put this into perspective, the density of matter inside a nucleus is several orders of magnitude larger than that of regular chemical elements. This means that even a small amount of such material would weigh an enormous amount. Although it is unlikely for such conditions to occur on Earth, we can observe quark matter in compact stars, which are estimated to have even higher densities.

Quantum Chromodynamics (QCD) is a theory that describes the strong interactions of particles at the microscopic level. Baryons, which include protons and neutrons, are composed of strongly interacting quarks. QCD also describes the properties of dense baryonic matter and quark matter. Quark confinement is a unique property of QCD, which states that quarks cannot exist as free particles but must combine with other quarks and antiquarks to form color-neutral hadrons.

When matter becomes dense enough, baryons begin to overlap and quarks are shared between neighboring baryons. As the density increases, quarks become deconfined over large distances, leading to the emergence of quark matter. This means that it is no longer appropriate to describe the matter in terms of hadrons, but rather in terms of quarks.

Transition from hadronic matter to deconfined quarks

This section of the essay will focus on the transition from a baryonic phase (neutron and proton phase) to a quark phase in neutron stars, based on the research of [1, 2]. The aim is to demonstrate that an increase in baryonic density can lead to conditions that favor a quark matter phase.

A phase transition involves a change or reaction that moves particles from one phase to another while conserving specific quantities. For example, the number of water molecules remains constant in a simple gas-liquid water transition, and each conserved quantity corresponds to a unique chemical potential. In the more complex phase transition from the baryonic phase to the free quark phase, two quantities are conserved: the baryon number and the electric charge, resulting in two distinct chemical potentials, the baryon chemical potential (μ_B) and the electric chemical potential (μ_Q). Chemical equilibrium is reached when both potentials are equal in both phases, and mechanical equilibrium must also be achieved, meaning that the pressure of both phases should be equivalent.

To add the chemical potential of conserved quantities to the system's Hamiltonian, the potentials of the conserved quantities, μ_B and μ_Q, are multiplied by the quantity of those quantities, N_B and N_Q, and then subtracted from the Hamiltonian. In this case, both the neutron and proton have a baryon number of one, and the electron has a baryon number of zero. The proton has a charge of plus one, the electron has a minus one, and the neutron has zero charge. The resulting equation is as follows:

$$\begin{aligned}
\mathcal{H}_{\text{Baryonic-phase}} &= \mathcal{H}_0 - \mu_B N_B - \mu_Q N_Q && (1)\\
&= \mathcal{H}_0 - \mu_B(N_p + N_n) - \mu_Q(N_p - N_e) && (2)\\
&= \mathcal{H}_0 - \mu_B N_p - \mu_B N_n - \mu_Q N_p + \mu_Q N_e && (3)
\end{aligned}$$

where $\mathcal{H}_0$ is the remaining part of the Hamiltonian (free or interacting) for the particles at hand. Collecting all the chemical potentials in front of the particle species we get,

$$\mathcal{H}_{\text{Baryonic-phase}} = \mathcal{H}_0 - (\mu_B + \mu_Q)N_p - \mu_B N_n + \mu_Q N_e \tag{4}$$

Sifting through the equation, we see that,

$$\begin{aligned}
\mu_n &= \mu_B && (5)\\
\mu_p &= \mu_B + \mu_Q && (6)\\
\mu_e &= -\mu_Q && (7)
\end{aligned}$$

and also,

$$\mu_B = \mu_n = \mu_p + \mu_e. \tag{8}$$

The last equality in the equation above can be justified by plugging in eq.(6), eq.(7) in eq.(8)

In the quark phase, we can start with similar reasoning. The main difference is that the quarks now have a baryon number of a third, the up quark has an electric charge of two-thirds, and the down quark has minus one-third. We can write down the Hamiltonian as,

$$\mathcal{H}_{\text{Quark phase}} = \mathcal{H}_0 - \mu_B\left(\frac{1}{3}N_u + \frac{1}{3}N_d\right) - \mu_Q\left(\frac{2}{3}N_u - \frac{1}{3}N_d - N_e\right) \tag{9}$$

Again, collecting all the chemical potentials in front of the particle species,

$$\mathcal{H}_{\text{Quark-phase}} = \mathcal{H}_0 - \left(\frac{1}{3}\mu_B + \frac{2}{3}\mu_Q\right)N_u - \left(\frac{1}{3}\mu_B - \frac{1}{3}\mu_Q\right)N_d + \mu_Q N_e \tag{10}$$

Carefully examining, we have,

$$\begin{aligned}
\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q\\
\mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q\\
\mu_e &= -\mu_Q.
\end{aligned} \tag{11}$$

From here, we can also deduce that,

$$\mu_B = \mu_u + 2\mu_d = 2\mu_u + \mu_d - \mu_e. \tag{12}$$

also, we can recognize the quark flavor transition

$$\mu_d = \mu_u + \mu_e \tag{13}$$

This is consistent with charge conservation because a down quark has a charge of a negative one-third, and the up quark has a two-thirds charge.

Taking into account that during the phase transition, both the phases will have equal Baryon chemical potential μ_B and electric charge chemical potential μ_Q, we get the following two expressions,

$$\begin{aligned} \mu_p &= 2\mu_d + \mu_u \\ \mu_n &= \mu_u + 2\mu_d \end{aligned} \tag{14}$$

This is very well consistent with the well-known two facts; a proton comprises two up and one down quark, and a neutron comprises one up and two down quarks.

The equation of state of the baryonic phase is well studied and understood from the TOV equations for a neutron star [1]. The next natural step is to work out the equation of state for the quark phase. Working under the assumption that such a phase only exists at high densities, we can work in the limit where all the quark masses are negligible (we will also consider the electron to be massless in this limit). To proceed, we first must figure out the energy density and pressure of an ideal gas of massless quarks. The major difference between this and what we have seen in standard textbooks is that quarks have a color charge that is generated by the $SU(3)$ symmetry of Quantum Chromodynamics.

Ideal massless quark gas By utilizing the Fermi distribution, which behaves as a step function at zero temperature, we can express the density of particles using the Fermi momentum. Assuming that fermions are free particles, their state is unaffected by degrees of freedom such as spin and color. Consequently, the particle density can be obtained by multiplying the total number of these degrees of freedom g by the momentum integral over the Fermi distribution. Therefore, we obtain the following equation:

$$n = g\int_0^\infty \frac{d^3k}{(2\pi)^3}\Theta(k_F - k) = \frac{gk_F^3}{6\pi^2} \tag{15}$$

[1] In contrast to what I had planned in the summary, I decided to skip going the TOV route because it was becoming too much General Relativity and Astrophysics. Instead I decided to focus on more statistical mechanics results

Solving for k_F which is the *Fermi momentum* we get the following expression,

$$k_F = \left(\frac{6\pi^2 n}{g}\right)^{\frac{1}{3}} \tag{16}$$

For a massless particle, the dispersion relation is given by $E(k) = k$ (we are working in units where $\hbar = c = 1$), the Fermi energy is given by,

$$E_F = k_F = \left(\frac{6\pi^2 n}{g}\right)^{\frac{1}{3}} \tag{17}$$

This Fermi energy equals the chemical potential $E_F = \mu$. Next, to get the energy density ρ, we will perform a momentum integral over the Fermi distribution multiplied by the energy density. Hence,

$$\rho = g\int \frac{d^3k}{(2\pi)^3} E(k) = \frac{g}{(2\pi)^3}\int_0^{k_F} \left(\int d\Omega\right) k^2 E(k) dk \tag{18}$$

$$= g\frac{4\pi}{(2\pi)^3}\int_0^{k_F} k^3 dk = \frac{g}{8\pi^2}k_F^4 = \frac{3}{4}\left(\frac{6\pi^2}{g}\right)^{\frac{1}{3}} n^{\frac{4}{3}} = \frac{3}{4}\frac{g\mu^4}{6\pi^2} \tag{19}$$

$$\therefore \ \rho = \frac{g\mu^4}{8\pi^2} \tag{20}$$

Mathematical interlude : Derivation

We can quickly derive an expression for pressure in terms of energy density ρ, the particle density n, and the Fermi energy E_F. Starting with the following expression,

$$P = -\frac{\partial E}{\partial V} = -\frac{\partial(\rho V)}{\partial V} \tag{21}$$

$$= -V\frac{\partial \rho}{\partial V} - \frac{\partial V}{\partial V}\rho \tag{22}$$

$$= -V\frac{\partial \rho}{\partial n}\frac{\partial n}{\partial V} - \rho \tag{23}$$

The two partial derivatives give us ($E(k)$ is the dispersion relation of the particle species in question),

$$\frac{\partial \rho}{\partial n} = \frac{\partial}{\partial n}\left(g\int \frac{d^3k}{(2\pi)^3}E(k)\right) = \frac{g}{2\pi^2}\frac{\partial}{\partial n}\left(\int_0^{k_F} k^2 E(k) dk\right) \tag{24}$$

$$= \frac{g}{2\pi^2}\frac{\partial k_F}{\partial n}\frac{\partial}{\partial k_F}\left(\int_0^{k_F} k^2 E(k) dk\right) \tag{25}$$

$$= \frac{g}{2\pi^2} \frac{\partial k_F}{\partial n} k_F^2 E_F \tag{26}$$

$$\frac{\partial k_F}{\partial n} = \frac{\partial}{\partial n} \left(\frac{6\pi^2 n}{g} \right)^{\frac{1}{3}} = \left(\frac{6\pi^2}{g} \right)^{\frac{1}{3}} \frac{\partial}{\partial n} n^{\frac{1}{3}} = \left(\frac{6\pi^2}{g} \right)^{\frac{1}{3}} \frac{1}{3n^{\frac{2}{3}}} \tag{27}$$

Plugging in the expression for n from eq.(15), we get,

$$\frac{\partial k_F}{\partial n} = \left(\frac{6\pi^2}{g} \right)^{\frac{1}{3}} \frac{1}{3} \left(\frac{6\pi^2}{g k_F^3} \right)^{\frac{2}{3}} = \left(\frac{2\pi^2}{g} \right) \frac{1}{k_F^2} \tag{28}$$

$$\therefore \; \frac{\partial \rho}{\partial n} = E_F \tag{29}$$

The second partial derivative in eq.(23) is,

$$\frac{\partial n}{\partial V} = \frac{\partial}{\partial V} \frac{N}{V} = -\frac{N}{V^2} = -\frac{n}{V} \tag{30}$$

Together we get the following equation,

$$P = \mu n - \rho \tag{31}$$

where $\mu = E_F$ has been assumed.

The pressure can be computed by plugging in

$$\rho = \frac{g}{8\pi^2} k_F^4 \tag{32}$$

into the relation derived in the box above,

$$P = \mu n - \rho = \frac{g k_F^4}{6\pi^2} - \frac{g k_F^4}{8\pi^2} = \frac{g}{24\pi^2} k_F^4 \tag{33}$$

These relations ρ, P hold for both quarks and high-energy neutrons and electrons. The degrees of freedom g is six for quarks (two for spin and three for color)—for neutrons and electrons $g = 2$ coming from the spin degree of freedom.

We are now in a state to express the chemical potential ($\mu = E_F$) in terms of their number density,

$$P + \rho = \mu n \tag{34}$$

$$\frac{g}{6\pi^2} k_F^4 = \mu n \tag{35}$$

$$\frac{g}{6\pi^2}\left(\frac{6\pi^2 n}{g}\right)^{\frac{4}{3}} = \mu n \tag{36}$$

$$\therefore\ \mu = \left(\frac{6n}{g}\right)^{\frac{1}{3}} \pi^{\frac{2}{3}} \tag{37}$$

Using this, we have the following three crucial equations for the quarks ($g = 6$) and electrons ($g = 2$),

$$\mu_u = \pi^{\frac{2}{3}} n_u^{\frac{1}{3}} \tag{38}$$

$$\mu_d = \pi^{\frac{2}{3}} n_d^{\frac{1}{3}} \tag{39}$$

$$\mu_e = (3\pi^2)^{\frac{1}{3}} n_e^{\frac{1}{3}} \tag{40}$$

Using eq.(11), we can represent the number density of these quarks and electrons as a function of the Baryon and electric chemical potential,

$$n_u = \frac{1}{\pi^2}\left(\frac{1}{3}\mu_B + \frac{2}{3}\mu_Q\right)^3 \tag{41}$$

$$n_d = \frac{1}{\pi^2}\left(\frac{1}{3}\mu_B - \frac{1}{3\mu_Q}\right)^3 \tag{42}$$

$$n_e = -\frac{1}{3\pi^2}\mu_Q^3 \tag{43}$$

Imposing electric neutrality in the quark phase, we get,

$$n_e = \frac{2}{3}n_u - \frac{1}{3}n_d \tag{44}$$

$$-\frac{1}{3}\mu_Q^3 = \frac{2}{3}\left(\frac{1}{3}\mu_B + \frac{2}{3}\mu_Q\right)^3 - \frac{1}{3}\left(\frac{1}{3}\mu_B - \frac{1}{3}\mu_Q\right)^3 \tag{45}$$

Solving this equation to express μ_Q in terms of μ_B using Mathematica, we get the following as the only real solution,

$$\mu_Q = -7.23 \times 10^{-2}\ \mu_B \tag{46}$$

The electric chemical potential (μ_Q) measures the energy needed to add an electrically charged particle to the system, such as an electron or a quark. The electric charge density of the system is related to the value of μ_Q.

In a neutron star environment, where both baryonic matter and electrically charged particles such as electrons are present, the electric charge density affects the system's overall energy. The system's energy is also high when the charge density is high.

Eq.(46) implies that as the baryonic chemical potential (μ_B) increases, the electric chemical potential (μ_Q) decreases. This means that the electric charge density of the system also decreases as the baryonic density increases. As a result, the system's energy decreases, making the quark phase more favorable.

The quark phase is more favorable when the system's energy is lower than the baryonic phase's. In the quark phase, quarks are deconfined and can move freely, leading to a decrease in the system's overall energy. Therefore, when the system's electric charge density decreases, the system's energy decreases, making the quark phase more favorable.

Overall, the relationship between the baryonic and electric chemical potentials affects the system's energy and the system phase behavior. Eq.(46) suggests that as the baryonic density increases, the electric charge density decreases, leading to a decrease in the system's overall energy and making the quark phase more favorable.

2 Cooper instability and Color Superconductivity

We have established in the previous section that theoretically speaking, at arbitrarily high densities, we can have quark matter. For neutron stars the chemical potential can go upto $\mu \approx 400$ MeV [3]. At these number, we are in a phase comprised of quark matter, we will establish a few properties for the same. In particular, we will argue that quark matter is a color superconductor.

We recall that Fermions obey the Pauli exclusion principle, i.e., no more than two can occupy the same state. Regarding phase space, the position variables are occupied up to the brim in a finite-sized volume (for example, in the core of a compact astrophysical object like neutron stars). Hence, to keep adding more Fermions to the same volume, the particles need to go higher up the momentum ladder. If the Fermions are non-interacting, we will have a Fermi sea of filled states. This corresponds to filling all the states with energy less than the Fermi energy $E_F = \mu$, and all states above E_F are empty. Mathematically speaking, at zero temperature and sufficiently high density, this corresponds to,

$$f_F(k) = \Theta(\mu - E(k)) \tag{47}$$

All the states up to that with a Fermi momentum of $k_F = \sqrt{\mu^2 - m^2}$ are occupied, and the quarks form a spherical Fermi surface in momentum space. Another important result to keep in mind is from [4, 3] is that in the chiral limit (masses of the quarks tend to zero), the pressure of the zero temperature free quark matter is given by,

$$P^{(0)} = 2N_f N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\mu - E_{\mathbf{k}}\right) \theta\left(\mu - E_{\mathbf{k}}\right) - B = \frac{N_f N_c}{12\pi^2}\mu^4 - B \tag{48}$$

where the overall factor $2N_f N_c$ counts the total number of degenerate quark states, namely 2 spin states (i.e., s = ±1/2), N_f flavor states (e.g., up, down and strange), and N_c color states (e.g., red, green and blue). The extra term B in eq.(48) , called the bag constant, was added by hand. This term effectively assigns a nonzero contribution to the vacuum pressure and, in this way, provides the simplest modelling of the quark confinement in QCD [4, 5]. By equating $P^{(0)}$ to zero we can see that at the vacuum pressure higher is than the quark matter when,

$$\mu < \left(\frac{4\pi^2 B}{N_f}\right)^{\frac{1}{4}} \tag{49}$$

At this moment we are not considering any charge neutrality or β-equilibrium in quark matter (This is necessary in Neutron stars and we use these conditions when we derive a particular result in the next section). The energy density of such quark matter is just a simple calculation away and is given by,

$$\epsilon^{(0)} \equiv \mu \frac{\partial P^{(0)}}{\partial \mu} - P^{(0)} = \frac{N_f N_c}{4\pi^2} \mu^4 + B \tag{50}$$

where we are now using ϵ^0 to denote the energy density.

Only the quarks at the Fermi surface can interact and exchange momenta (illustrated in fig.(1). This means any attractive interaction between the Fermions will cause pairing instability of the Fermi surface, i.e., the Fermions on the edge of the surface pair up due to this attraction and start forming Cooper pairs. The macroscopic formation of such cooper pairs is called a condensate of cooper pairs. This is the famous Cooper instability [6, 7].

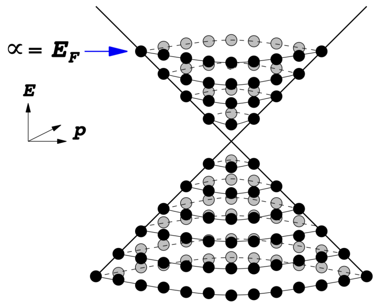

Figure 1: Illustration of a filled Fermi sea. (Figure borrowed from [8] for this essay.)

Intuitively, we can understand this phenomenon in the following way: The core idea of Cooper instability comes from the fact that the system tries to minimize its free energy,

$$F = E - \mu N \tag{51}$$

The Fermi surface is, by definition, $E_F = \mu$. If we ignore any attractive interaction, we can see that free energy is minimized at $E = E_F$. Hence, adding or removing a single particle costs zero free energy - as adding the particle costs energy E_F, which in turn increases N by one, keeping F unchanged (again, this is true only under the assumption that the particles have no interaction).

If we turn on (even a weak) attractive interaction, it still costs no free energy to add a pair of particle (or holes - we say that removing a particle from the Fermi sea create a hole) close to the Fermi surface. On the contrary, the attractive attraction between such pairs lowers the system's free energy. As mentioned before, our system tries to minimize its free energy, which favors the formation of such pairs. These pairs are bosonic and will form a condensate. The ground state will be a superposition of states with all numbers of pairs, breaking the fermion number symmetry.

Comment : Superconductivity of electrons

The repulsion between electrons is caused by their similar charges in QED. However, in a metal with a lattice of positively charged ions, we can utilize phonons to create attractive interactions between electrons, resulting in Cooper pairing. This pairing is delicate and easily disrupted by thermal fluctuations, which explains why metals only exhibit superconductivity at very low temperatures.
The resulting Cooper pair condensate has a charge density that can interact with the electromagnetic field. This interaction causes the electromagnetic field to behave differently from its behavior in a vacuum, leading to an effective mass for the photon via the Higgs mechanism. This mechanism arises from the breaking of electromagnetic gauge symmetry by the condensate. As a result of this symmetry breaking, a massless Goldstone boson is produced and absorbed by the photon to give it a mass. This massive photon, also known as a Meissner-Ochsenfeld photon, is responsible for the Meissner effect, the expulsion of magnetic fields from a superconductor.

Superconducting Quark Matter

We have motivated the fact that an attractive interaction between quarks drives Cooper instability. Here we quickly summarize where such an attractive interaction originates. The QCD Lagrangian for ψ_i^a quark fields ($a \in \{1,2,3\}$ is the color index and $i \in \{1,2\}$ is the flavor index) reads,

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i^a \left(i\gamma^\mu \partial_\mu + \gamma^0 \mu - m_i^{(0)} \right) \psi_i^a + g A_\mu^A \bar{\psi}_i^a \gamma^\mu T_{ab}^A \psi_i^b - \frac{1}{4} G_{\mu\nu}^A G^{A,\mu\nu}. \tag{52}$$

Here, A_μ^A is the vector gauge field in $\mathrm{SU}(3)_c$ where $G_{\mu\nu}^A = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + g f^{ABC} A_\mu^B A_\nu^C$ is the field strength, and the generators of the color transformations are defined as $T_{ab}^A = \frac{1}{2}\left(\lambda^A\right)_{ab}$ where λ^A are the Gell-Mann matrices. The current quark masses and the quark chemical potential are denoted by $m_i^{(0)}$ and μ , respectively.

Comment [4]

At high densities, one can neglect small current masses of quarks. Then, the QCD Lagrangian density becomes invariant under $SU(2)_L \times SU(2)_R$ global chiral transformations.In QCD, the constituent masses of quarks m_i are generated dynamically, and they can be very different from the current masses $m_i^{(0)}$, appearing in the Lagrangian density. At zero density, for example, typical values of the constituent quark masses are of order $\frac{1}{3}m_n \approx 313$ MeV even in the chiral limit. At high densities, on the other hand, the masses of the up and down quarks become small. This is because they are proportional to the value of the choral condensate $\langle \bar{\psi}_L \psi_R \rangle$ which melts in dense matter. Thus, it is often also justified to neglect the constituent quark masses in studies of color superconducting phases.

All of this brings context to the fact that, for a quark-quark scattering amplitude in a one-gluon exchange is proportional to the following color tensor :

$$\sum_{A=1}^{N_c^2-1} T^A_{aa'} T^A_{b'b} = -\frac{N_c+1}{4N_c}\left(\delta_{aa'}\delta_{b'b} - \delta_{ab'}\delta_{a'b}\right) + \frac{N_c-1}{4N_c}\left(\delta_{aa'}\delta_{b'b} + \delta_{ab'}\delta_{a'b}\right) \quad (53)$$

(The interaction amplitude in this channel is essentially proportional to the Quantum Electrodynamics (QED) amplitude by photon exchange multiplied by the SU(3) group theoretic factor) This part of the interaction tells us how the amplitudes change as a function of the color of the incoming a, a' and outgoing b, b' particles. The first term is antisymmetric in the interchange of color indices of the incoming quarks (and those of the outgoing quarks). It corresponds to the attractive color anti-triplet channel, while the second symmetric term corresponds to the repulsive sextet channel. When $\mu \gg m$, the typical momentum scale of quarks is μ. When $\mu \gg \Lambda_{\text{QCD}}$, QCD becomes weakly coupled, and quark-quark interaction is dominated by a single gluon exchange enabling the attractive nature of the first term of the color tensor (fig.(2)).

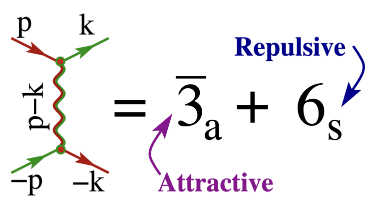

Figure 2: The diagrammatic representation of the one-gluon exchange interaction between two quarks in QCD. The color structure of the corresponding amplitude contains an antisymmetric antitriplet and a symmetric sextet channel. (Figure and caption borrowed from [4] for this essay.)

At temperatures small compared to the Fermi energy, $T_F \ll E_F$, most of the quarks will reside under the Fermi surface, and a few are thermally excited, creating holes inside the Fermi sea. The cost of creating quarks having Fermi momentum is small, and we expect Cooper instability due to the attractive interaction as described in eq.(53). This instability will persist up to a critical temperature T_c above which we will not have instability. The instability, just like in BCS superconductors, can be spontaneously resolved by modifying the dispersion relation by creating a gap in the energy of the excitation spectrum. The excitation spectrum with this gap of a superconducting particle looks like this,

$$E(k) = \sqrt{(\epsilon(k) - \mu)^2 + \Delta^2} \tag{54}$$

where $\epsilon(k) = \sqrt{\vec{k}^2 + m^2}$ is the regular dispersion of a particle of mass m. Δ is called the gap in the energy spectrum, and for the special case that $\Delta = 0$, the particle is said to be ungapped. If the gap exists, then these particles possess a finite amount of energy even at the Fermi surface, precisely $E(k_F) = \Delta$. This makes the free energy cost of creating a pair of gapped quarks at the Fermi surface non-zero, i.e., 2Δ (factor of two as we are creating a pair), which removes the instability. This is illustrated in fig.(3)

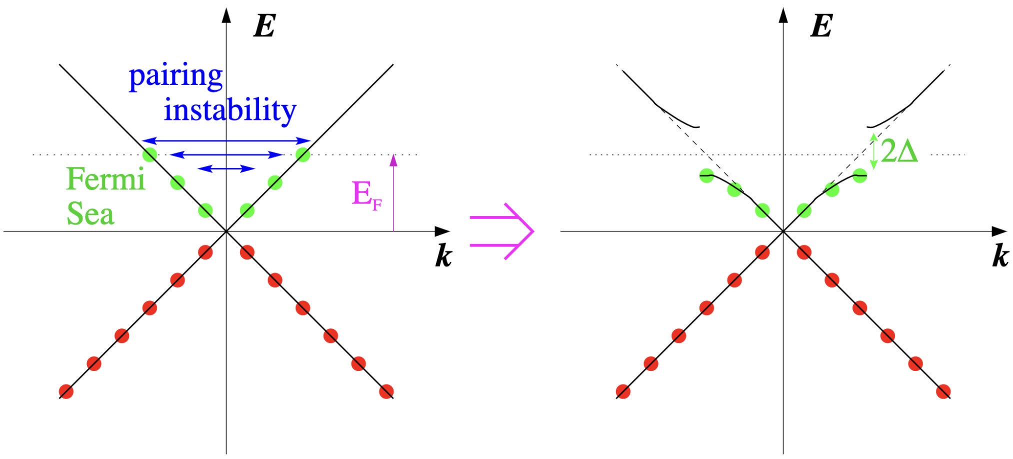

Figure 3: Pairing instabilities leading to superconductivity. (Figure borrowed from [9] for this essay.)

As opposed to electrons in BCS superconductivity, quarks come in various flavors (as mentioned before, we focus on up, down, and strange as the other three quarks, top, bottom, and charm, are too heavy to be included as our chemical potential does not allow for these quarks to be a part of the quark matter we are discussing, i.e., low temperature, high-density quark matter). Having three flavors of quarks at our disposal, many phases of Color SuperConductivity (CSC)[2] can be theorized.

[2]Why is it called *color* superconductivity? Because the gluons that enable these Cooper pairings have a color charge under the QCD SU(3) gauge group - just like electric charge is the electric charge of the U(1) electromagnetic gauge group. And the different types of pairings depending on the color of the quarks

Typical superconducting gaps in quark matter at densities of relevance to neutron stars suggest that $\Delta \simeq 20 - 100$ MeV when $\mu = 400$ MeV[10, 11] [3].

The attractive potential for the quarks in the anti-symmetric (color-triplet) channel (like $(r_1 b_2 - b_1 r_2)$) has a potential given by $V^A_{qq} = -2\alpha_s/3r$ (this can easily found in a good standard model textbook like [13]). The attraction in the triplet channel can result in s-wave pairing between quarks in the spin-zero and spin-one channels. Explicit calculations show that the pairing energy, Δ, is especially large for the spin-zero case. This type of pairing can only occur between unlike flavors of quarks to ensure that the diquark pair (a boson) has a symmetric wavefunction.

What does it mean mathematically to have a Cooper pair? It means that the two particles that form a pair are correlated. This correlation function can be measured in terms of a particle-particle expectation value. More precisely, when the expectation value of two particles is non-zero, there is a pairing. In terms of equations and our context, this means [4]

$$\langle q^{\alpha}_{ia} q^{\beta}_{jb} \rangle = \Delta \neq 0 \tag{55}$$

where $\alpha, \beta \in \{r, g, b\}$ is the color index, $i, j \in \{u, d, s\}$ is the flavor index and $a, b \in \{\uparrow, \downarrow\}$ is the spin index. This gives a total of 9×9 matrix of possible Cooper pairing patterns.

The attractive channel from which we form quark pairs is overall antisymmetric. This anti-symmetry has three parameters: color, spin, and flavor. By construction, the color tensor is antisymmetric in color and spin. Hence, these pairings formed due to the attractive part of the color tensor must be flavor antisymmetric too. This means that pairing different flavors is preferred.

Color Flavor Locked phase (CFL)

Let's summarize[5] the simplest case where all three quarks are massless in the limit $\mu \gg m_u, m_d, m_s$. There are three colors and three flavors, and this

[3] The calculation for these gaps is an intricate exercise in Quantum Field Theory (QFT). An excellent summary of how these techniques are used to find the gap equation (which, as the name suggests, gives us an expression for gaps in different CSC models) can be found in [4]. Detailed calculations can be found in [2, 12]

[4] The fact that the gap Δ is directly connected to such a correlation function (or expectation value) can be computed by QFT techniques.

[5] Deriving the results in this section would take us much beyond the 20 page limit. Instead I have tried my best to condense the main facts of such a phase.

equivalence gives a special pairing pattern [14] [6],

$$\left\langle q_i^\alpha q_j^\beta \right\rangle \sim \Delta(\delta_i^\alpha \delta_j^\beta - \delta_j^\alpha \delta_i^\beta) = \Delta(\epsilon^{\alpha\beta n}\epsilon_{ijn}) \tag{56}$$

The possibility of all the pairings is shown in fig.(4)

$\tilde{Q}$	0	0	0	-1	$+1$	-1	$+1$	0	0
	u	d	s	d	u	s	u	s	d
u		Δ	Δ						
d	Δ		Δ						
s	Δ	Δ							
d					$-\Delta$				
u				$-\Delta$					
s							$-\Delta$		
u						$-\Delta$			
s									$-\Delta$
d								$-\Delta$	

Figure 4: Possbility of all Cooper pairings in CFL phase (Figure borrowed from [8] for the purpose of this essay).

This kind of pairing is called Color Flavor Locked (CFL) phase. The Kronecker delta functions link the color and flavor indices. The intricate meaning hidden behind this is that such a condensate transforms non-trivially under color and flavor transformations. But, they remain invariant if you simultaneously rotate both flavor and color, i.e. these symmetries are locked together.

Another important feature of such a phase is that it breaks Chiral symmetry. The particular structure of symmetry breaking is as follows :

$$[SU(3)_c] \times SU(3)_R \times SU(3)_L \times U(1)_B \to SU(3)_{c+L+R} \times \mathbb{Z}_2 \tag{57}$$

One of the generators of SU$(3)_{L+R}$ is the electric char g_e, which generates the U$(1)_Q$ gauge symmetry . This means,

$$SU(3)_{L+R+c} \supset \left[U(1)_{\tilde{Q}}\right] \tag{58}$$

which is unbroken and correspond to a simultaneous electromagnetic and color rotation. Seven gluons and one gluon-photon linear combination become massive via the Meissner effect. The mixing angle is,

$$\cos\theta = \frac{g}{\sqrt{g^2 + 4e^2/3}}, \quad e \ll g \to \theta \sim 0 \tag{59}$$

[6] An excellent paper and a fun read. But the derivation of the following fact again uses many QFT techniques. If I had even attempted to summarize the whole procedure, I would have needed at least a few pages of this essay. Instead, I focused on the results and talked more about statistical mechanics overall.

The $\bar{Q}$ photon is the original photon with a small mixture of gluon. Interestingly, the $\bar{Q}$ electromagnetic fields satisfy the Maxwell's equation. Using that, one can compute the index of refraction to be [],

$$n = 1 + \frac{e^2 \cos\theta^2}{9\pi^2}\left(\frac{\mu}{\Delta_{CFL}}\right)^2 \tag{60}$$

This proves that the CFL phase is a transparent insulator. The massive gluons make it a color superconductor (Just like the massive photons in the case of normal matter BCS theory are responsible for superconductivity).

In order to write down the expression for the pressure of three-flavor quark matter in the CFL phase, we take the Pauli pressure contributions of nine (three flavors times three colors) quarks and add the correction due to color superconductivity (i.e. pairing at the Fermi surface also contributes to the pressure). The result of doing this is given in [4],

$$P_{(\mathrm{CFL})} \simeq \frac{3\mu^4}{4\pi^2} - B + 3\frac{\mu^2\Delta^2}{\pi^2} \tag{61}$$

and the energy density is given by [4],

$$\epsilon_{(\mathrm{CFL})} \simeq \frac{9\mu^4}{4\pi^2} + B + 3\frac{\mu^2\Delta^2}{\pi^2}\left(1 + \frac{2\mu}{\Delta}\frac{\partial\Delta}{\partial\mu}\right) \tag{62}$$

We can clearly see that we have an additional term corresponding to Δ in comparison to eq.(48), eq.(50) which correspond to the free quark matter. These two expressions give a parametric representation of the equation of state of dense quark matter in the CFL phase. This is a clear indication that the thermodynamic properties have been altered due to the presence of a non-zero gap.

2-SuperConductor phase (2SC)

In this section we summarize another case where $m_s > \mu > m_u, \mu_d$.

The main gist of such a phase is well described in [15] and goes as follows : It is a reasonable approximation to neglect the u and d quark masses at densities of relevance to neutron stars where $\mu \sim 400$ MeV. The strange quark mass $m_s \sim 200$ MeV, on the other-hand, cannot be neglected. The difference in Fermi momenta between light quarks and the strange quark is $m_s^2/2\mu$. Thus, when $\Delta < m_s^2/2\mu$, pairing involving strange quarks will be suppressed (We derive something similar to the two previous points made this in the next section). In the limit of innate strange quark mass, i.e, in their absence, only light quarks pair. This phase is called 2SC (two-flavor superconductor) and it is also characterized by pairs that are antisymmetric in flavor.

Conjectured QCD phase diagram

The famous conjectured QCD phase diagram is shown in fig.(5). We can see that the CSC phases as discussed are in the low-temperature, high chemical potential region.

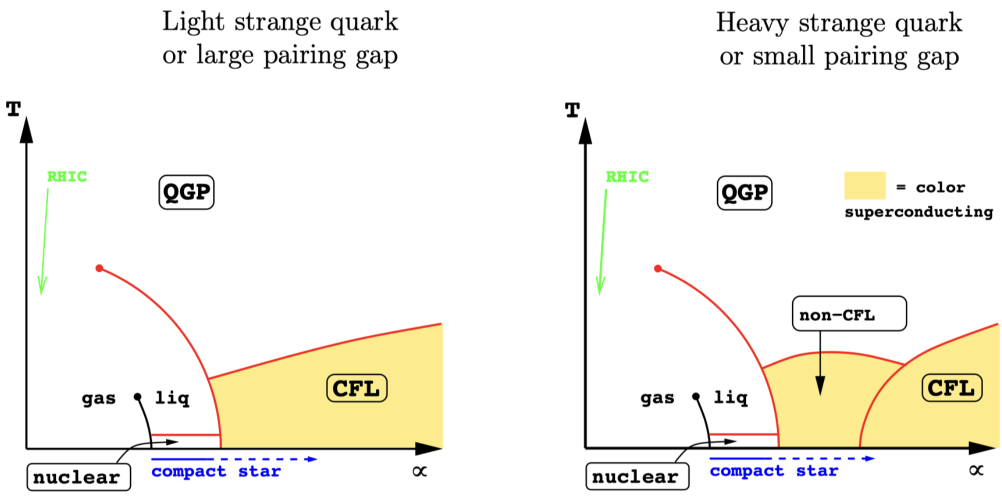

Figure 5: Conjectured phase diagram for QCD (Left: Light strange quark, Right: Heavy strange quark) (Figure borrowed from [16] for the purpose of this essay).

The main difference between the left and the right diagram is that, when we assume the strange quark to be heavy, we end up having a phase where we are still color-superconducting but not CFL. In the next section we discuss in detail the realistic scenario for quark matter in neutron stars where the strange quark is massive.

3 Color Superconductivity in compact stars

For this section we will follow parts of [17] and do many of the calculations explicitly. As established in the previous sections, compact stars might be host to this unique phenomenon of color superconductivity (as described in the previous section) due to the possibility of the abundance of quark matter (as described in the section 1. Starting with the fact that BCS critical temperature is given by,

$$T_c = 0.57\,\Delta_{\mathrm{BCS}} \tag{63}$$

In the previous section, we cited the fact that for QCD, the range for Δ_{BCS} is expected to be in between 20 to 100 MeV.

In the previous section, we cited results showing that QCD favors the formation of BCS condensates in the ideal cases of two or three massless flavors of quarks (2SC and CFL). Realistically, we must consider that the quarks have different masses and Fermi energies/momenta.

To obtain an order of magnitude estimation for color superconductivity in neutron stars, we will begin by examining a free Fermi gas composed of three flavors of quarks, namely the up, down, and strange quarks. In contrast to our previous analysis in Section 1, where we only considered the ideal scenario of massless up and down quarks, this analysis considers a more realistic scenario that includes the strange quark with a mass. Our chemical potentials allow for the inclusion of the strange quark without any bias. However, we must note that heavier quarks are unlikely, as they cannot be created due to the upper bound of our chemical potential.

We assume that weak interactions are in equilibrium. Let μ_0 be the average chemical potential given by,

$$\mu_0 = \frac{1}{3}\left(\mu_u + \mu_d + \mu_s\right). \tag{64}$$

Using this, we write our chemical potentials as follows,

$$\mu_u = \mu_0 - \frac{2}{3}\mu_e \tag{65}$$

$$\mu_d = \mu_s = \mu_0 + \frac{1}{3}\mu_e \tag{66}$$

and the Fermi energy/momenta are given by,

$$E_{F,u} = k_{F,u} = \mu_u \tag{67}$$

$$E_{F,d} = k_{F,d} = \mu_d \tag{68}$$

$$E_{F,s} = k_{F,s} = \sqrt{\mu_s^2 - m_s^2} \tag{69}$$

Similar to our previous calculations, assuming electric neutrality, we can write down the following equation

$$\mu_u n_u + \mu_d n_d + \mu_s n_s + \mu_e n_e = \mu_0 n_q - \mu_e Q \tag{70}$$

where

$$n_q = n_u + n_d + n_s \tag{71}$$

$$Q = \frac{2}{3}n_u - \frac{1}{3}(n_d + n_s) - n_e \tag{72}$$

in which the Q variable takes care of the fact that u has an electric charge of $\frac{2}{3}$, d and s have an electric charge of $-\frac{1}{3}$ and the electron has an electric charge of -1. To maintain electrical neutrality, $Q = 0$ is essential. This gives us the following equation,

$$n_e = \frac{\mu_e^3}{3\pi^2} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s \tag{73}$$

Comment

Starting from eq.(73), we will come to an well motivated approximate expression for μ_e in terms of m_s and μ_0.

$$n_e = \frac{2}{3\pi^2}\mu_u^3 - \frac{1}{3\pi^2}\mu_d^3 - \frac{1}{3\pi^2}\left(\mu_s^2 - m_s^2\right)^{\frac{3}{2}} \tag{74}$$

$$= \frac{2}{3\pi^2}\mu_u^3 - \frac{1}{3\pi^2}\mu_d^3 - \frac{1}{3\pi^2}\left(\mu_s^2 - m_s^2\right)^{\frac{3}{2}} \tag{75}$$

$$= \frac{2\mu_u^3}{3\pi^2} - \frac{1}{3\pi^2}\left[\mu_d^3 + \mu_s^3\left(1 - \frac{3m_s^2}{2\mu_s^2} + \frac{3m_s^4}{8\mu_s^4} + \mathcal{O}\left(\frac{m_s^6}{\mu_s^6}\right)\right)\right] \tag{76}$$

We made a Taylor expansion from the second to the third line. After making a substitution $\mu_d \to \mu_s$ and plugging in eq.(66) and making the manipulations as shown in the pages before the references and collecting the appropriate terms, we are left with

$$\mu_e \simeq \frac{m_s^2}{4\mu_0} \tag{77}$$

The electron chemical potential is small compared to the quark chemical potentials, and N_e is assumed to vanish while deriving the equation above.

The chemical potential for electrons is fixed by requiring electric neutrality. This exact statement corresponds to the following condition at zero temperature,

$$Q = \frac{\partial \Omega}{\partial \mu_e} = 0 \tag{78}$$

For each of the Fermionic species, the free energy Ω is given by,

$$\Omega = \frac{1}{\pi^2} \int_0^{k_F} k^2 (E(k) - \mu) dk \tag{79}$$

We have four fermions u, d, s and e, this gives us

$$\Omega = \frac{3}{\pi^2} \sum_{i=u,d,s} \int_0^{k_{F,i}} k^2 (E_i(k) - \mu_i) dk + \frac{1}{\pi^2} \int_0^{\mu_e} k^2 (k - \mu_e) dk \tag{80}$$

The dispersion relations for $m_u, m_d \ll m_s$ (we had assumed u, d to be massless to begin with anyway) are,

$$E_{u,d}(k) = k \tag{81}$$

$$E_s(k) = \sqrt{k^2 + m_s^2} \tag{82}$$

Plugging eq.(7), eq.(66), eq.(77) into Ω (and then into Mathematica), gives us,

$$\Omega \approx \frac{3}{4\pi^2} \mu^2 \left(m_s^2 - \mu^2 \right) - \frac{7 - 12 \log(m_s / 2\mu_0)}{32\pi^2} m_s^4 \tag{83}$$

Consequently, the baryon density is obtained as,

$$\rho_B = -\frac{1}{3} \frac{\partial \Omega}{\partial \mu} = \frac{1}{3\pi^2} \sum_{i=u,d,s} \left(p_F^i \right)^3 \approx \frac{\mu^3}{\pi^2} \left[1 - \frac{1}{2} \left(\frac{m_s}{\mu} \right)^2 \right] \tag{84}$$

The densities in the core are of the order of 10^{15}g cm^{-3}, which corresponds to a chemical potential of ~ 400 MeV

To go to baryon densities relevant to the central core of the star, i.e. densities from 6 to 8 times the nuclear matter density, one needs to go to higher values of μ and lower values of m_s where the difference among of the Fermi momenta is lower. This can be seen using the approximate expression that we derived for μ_e :

$$p_F^u \approx \mu - \frac{m_s^2}{6\mu}, \quad p_F^d \approx \mu + \frac{m_s^2}{12\mu}, \quad p_F^s \approx \mu - \frac{5m_s^2}{12\mu} \tag{85}$$

$$p_F^d - p_F^u \approx p_F^u - p_F^s \approx \frac{m_s^2}{4\mu} \tag{86}$$

This scenario in comparision to CFL and 2SC phase is very well illustrated in the fig.(6)

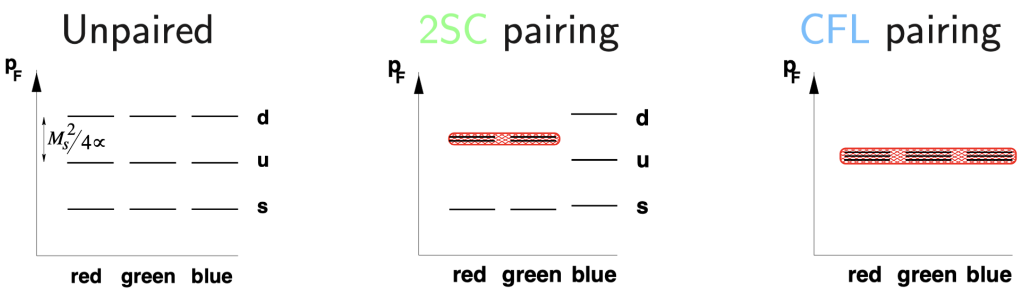

Figure 6: Comparison of Cooper pairing in different CSC phases. (Figure borrowed from [8] for the purpose of this essay).

This means that in the realistic scenario that where the strange quark is massive, CFL and 2SC are not preferred phase [7] In particular what we are observing right now is a mismatch in the Fermi-momentum of different particles. Such a mismatch can also motivate the formation of color condensates in the environment of compact stars due to the following reasons:

- Enhanced attractive interaction: Fermi momentum mismatch between different quark flavors can enhance the attractive interaction between the quarks, which can facilitate the formation of unconventional Cooper pairs. This is because the mismatch in the fermi momenta reduces the phase space available for the quarks to scatter, leading to a more attractive interaction mediated by gluon exchange. This can drive the formation of color superconducting or color superfluid phases, where the quarks form Cooper pairs with aligned momenta and spins.

- Energetic favorability: The formation of color condensates can be energetically favorable in the high-density environment of compact stars, where the quark-gluon plasma is expected to exist. At these densities, the strong interaction between the quarks can overcome the repulsive Coulomb force, leading to the formation of Cooper pairs and the condensation of color-charged objects. The presence of a fermi momentum mismatch can further enhance the energetic favorability of these exotic phases of matter, as the mismatch can reduce the kinetic energy of the quarks and allow them to form pairs with lower relative momenta.

- Robustness against perturbations: Color condensates can be more robust against perturbations than conventional superconductors, due to

[7] There are many other phases like CFLK^0, LOFF, etc. A collection of these can be found in [4, 15, 16, 17]

the topological protection of the Cooper pairs. In a color superconductor, the Cooper pairs have a non-trivial phase structure that can give rise to topological defects, such as vortices and domain walls. These defects can affect the transport properties and thermodynamics of the system, but they are also a signature of the unconventional pairing mechanism. The presence of a fermi momentum mismatch can further stabilize the topological defects, as the mismatch can lead to a non-uniform distribution of the Cooper pairs in momentum space, which can give rise to a richer variety of topological configurations.

Calculation A

In[1]:= `Clear["Global`*"]`

In[2]:= $\text{Series}\left[(1-x)^{\frac{3}{2}}, \{x, 0, 4\}\right]$

Out[2]= $$1-\frac{3x}{2}+\frac{3x^2}{8}+\frac{x^3}{16}+\frac{3x^4}{128}+O[x]^5$$

In[3]:= $$ne = \frac{2}{3\pi^2}\mu_u{}^3-\frac{1}{3\pi^2}\left(\mu_d{}^3+\mu_s{}^3\left(1-\frac{3}{2}\frac{m_s{}^2}{\mu_s{}^2}+\frac{3}{8}\frac{m_s{}^4}{\mu_s{}^4}\right)\right)$$

Out[3]= $$-\frac{\mu_d^3+\left(1+\frac{3 m_s^4}{8\mu_s^4}-\frac{3 m_s^2}{2\mu_s^2}\right)\mu_s^3}{3\pi^2}+\frac{2\mu_u^3}{3\pi^2}$$

In[4]:= $ne \mathbin{/.} \mu_d \to \mu_s$

Out[4]= $$-\frac{\mu_s^3+\left(1+\frac{3 m_s^4}{8\mu_s^4}-\frac{3 m_s^2}{2\mu_s^2}\right)\mu_s^3}{3\pi^2}+\frac{2\mu_u^3}{3\pi^2}$$

In[5]:= $$\%4 \mathbin{/.} \left\{\mu_s \to \left(\mu+\frac{1}{3}\mu_e\right), \mu_u \to \left(\mu-\frac{2}{3}\mu_e\right)\right\}$$

Out[5]= $$-\frac{\left(\mu+\frac{\mu_e}{3}\right)^3+\left(1+\frac{3 m_s^4}{8\left(\mu+\frac{\mu_e}{3}\right)^4}-\frac{3 m_s^2}{2\left(\mu+\frac{\mu_e}{3}\right)^2}\right)\left(\mu+\frac{\mu_e}{3}\right)^3}{3\pi^2}+\frac{2\left(\mu-\frac{2\mu_e}{3}\right)^3}{3\pi^2}$$

In[6]:= `Simplify[%]`

Out[6]= $$\frac{-27 m_s^4+12 m_s^2(3\mu+\mu_e)^2-16\left(27\mu^3\mu_e+\mu_e^4\right)}{72\pi^2(3\mu+\mu_e)}$$

In[7]:= $\text{Collect}[\%, m_s]$

Out[7]= $$-\frac{3 m_s^4}{8\pi^2(3\mu+\mu_e)}+\frac{m_s^2(3\mu+\mu_e)}{6\pi^2}-\frac{2\left(27\mu^3\mu_e+\mu_e^4\right)}{9\pi^2(3\mu+\mu_e)}$$

In[8]:= $$\text{Simplify}\left[-\frac{3 m_s^4}{8\pi^2(3\mu+\mu_e)}+\frac{m_s^2(3\mu+\mu_e)}{6\pi^2}-\frac{2\left(27\mu^3\mu_e+\mu_e^4\right)}{9\pi^2(3\mu+\mu_e)}\right]$$

Out[8]= $$\frac{-27 m_s^4+12 m_s^2(3\mu+\mu_e)^2-16\left(27\mu^3\mu_e+\mu_e^4\right)}{72\pi^2(3\mu+\mu_e)}$$

This expression is equal to $\frac{\mu_e{}^3}{3\pi^2}$. But, in comparison to all the other terms, this is small term. We can say that the above expression is equal to zero. Leaving us with,

$$-27 m_s^4+12 m_s^2(3\mu+\mu_e)^2-16\left(27\mu^3\mu_e+\mu_e^4\right) == 0;$$

Dividing by $m_s{}^4$ we have,

In[12]:= $$-27+12 m_s^{-2}(3\mu+\mu_e)^2-16 m_s{}^{-4}\left(27\mu^3\mu_e+\mu_e^4\right) == 0;$$

Calculation A

Getting rid of $O(1)$ parameters and taking $\mu >> \mu_e$, we get,

In[13]:= $12\, m_s^{-2}\, (3\,\mu)^2 - 16\, m_s^{-4}\, \left(27\,\mu^3\,\mu_e\right) == 0$

Out[13]=

$$\frac{108\,\mu^2}{m_s^2} - \frac{432\,\mu^3\,\mu_e}{m_s^4} == 0$$

In[20]:= $\text{Solve}\left[\frac{108\,\mu^2}{m_s{}^2} - \frac{432\,\mu^3\,\mu_e}{m_s{}^4} == 0,\, m_s\right]$

Out[20]=

$$\left\{\left\{m_s \to -2\,\sqrt{\mu}\,\sqrt{\mu_e}\right\},\,\left\{m_s \to 2\,\sqrt{\mu}\,\sqrt{\mu_e}\right\}\right\}$$

Taking the positive root, we get can have the expression, $\mu_e = \frac{m_s{}^2}{4\,\mu}$.

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