Problem. HZ production at a e^+e^- collider

Consider the process

$$e^-(p_1)e^+(p_2) \to h(k_1)Z(k_2) \tag{1}$$

with $q = p_1 + p_2$. The Feynman rule for $Zf\bar{f}$ -vertex is

$$-i\frac{g}{4\cos\theta_W}\gamma_\mu(V - A\gamma_5)\,, \qquad V = 2I_W^3 - 4Q\sin^2\theta_W, \quad A = 2I_W^3\,. \tag{2}$$

being $I_W^3 = \pm 1/2$ the isospin of the fermion and Q its electric charge. In the electron case ($f = e^-$):

$$V = -1 + 2\sin^2\theta_W\,, \qquad A = -1\,. \tag{3}$$

The Feynman rule for the hZZ vertex is

$$ig\frac{m_Z}{\cos\theta_W}g_{\mu\nu} = i\frac{2m_Z^2}{v}g_{\mu\nu}\,. \tag{4}$$

(a) Draw the Feynman diagram for the process and write down the amplitude of the process using the couplings mentioned above. Use unitary gauge.

(b) Write down the amplitude in terms of G_F by using

$$\frac{m_W}{m_Z} = \cos\theta_W, \qquad \frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}} \tag{5}$$

(c) Compute the unpolarised squared matrix-element $|\bar{M}|^2$, summed over final state spin and averaged over the initial polarizations, taking $m_e \to 0$

(d) Compute the differential cross section of the process in the C.O.M frame.

(e) Compute the total cross section of the above process. Check how the total cross section behaves with variation of m_h.

P.1. HZ production at a e^+e^- collider. (ROHAN KULKARNI)

Consider : $e^-(p_1)e^+(p_2) \to h(k_1)Z(k_2)$ with $q=p_1+p_2$.

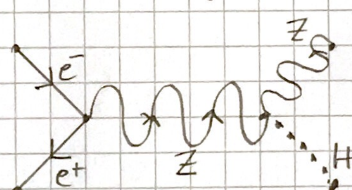
$Zf\bar{f}$ vertex : $-i\frac{g}{4\cos\theta_W}\gamma_\mu(V-A\gamma_5)$, $V=2I_W^3-4Q\sin^2\theta_W$ (Charge)

$A=2I_W^3$; $I_W=\pm\frac{1}{2}$ isospin of f.

For $(f=e^-)$: $V=-1+2\sin^2\theta_W$, $A=-1$

hZZ vertex : $ig\frac{m_Z}{\cos\theta_W}g_{\mu\nu} = i\frac{2m_Z^2}{v}g_{\mu\nu}$

(a) Draw a feynman diagram for the process & write down amplitude using couplings abv. Use unitary gauge.

$iM =$ 

$$= \bar{v}(p_2)\frac{-ig}{4\cos\theta_W}\gamma_\mu(V-A\gamma_5)u(p_1)\frac{-i}{k^2-m_Z^2}\left(g_{\mu\nu}-\frac{k^\mu k^\nu}{M_Z^2}\right)\frac{igm_Z}{\cos\theta_W}g_{\mu\nu}\epsilon^\nu(k_2)$$

(b) Write amplitude in terms of G_F using : $\frac{m_W}{m_Z}=\cos\theta_W$; $\frac{g^2}{8m_W^2}=\frac{G_F}{\sqrt{2}}$

$\to$ $M_W=\sqrt{\frac{\sqrt{2}}{8G_F}}\,g$ $\Rightarrow$ $\frac{1}{\cos\theta_W}=\sqrt{\frac{8G_F}{\sqrt{2}}}\frac{m_Z}{g}$

$$\therefore\ iM=\bar{v}(p_2)\frac{1}{4}\frac{8G_F}{\sqrt{2}}m_Z^3\gamma_\mu(V-A\gamma_5)u(p_1)\frac{-i}{k^2-m_Z^2}\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{M_Z^2}\right)g_{\mu\nu}\epsilon^\nu(k_2)$$

$$=\bar{v}(p_2)\sqrt{2}\,G_F m_Z^3\gamma_\mu(V-A\gamma_5)u(p_1)\frac{-i}{k^2-m_Z^2}\left(-g^{\mu\nu}+\frac{k^\mu k^\nu}{M_Z^2}\right)g_{\mu\nu}\epsilon^\nu(k_2)$$

$$=\bar{v}(p_2)\sqrt{2}\,G_F M_Z^3\gamma_\mu(V-A\gamma_5)u(p_1)\frac{-i}{k^2-M_Z^2}\left(\frac{k^2}{M_Z^2}-4\right)\epsilon^\mu(k_2)$$

(c) Compute $|\bar{M}|^2$: Summed over final spin state & Avg over init. polar^{n}.

$m_e\to 0$

$$\to\ |M|^2=\frac{G_F^2}{2(s-m_Z^2)^2}m_Z^6\sum_{s,s'}\left[\bar{v}_s\gamma_\mu(V-A\gamma_5)u_{s'}\right]\left(\sum_p\epsilon_p^\mu\epsilon_p^\nu\right)\left(\frac{k^2}{M_Z^2}-4\right)\left[\bar{u}_{s'}\gamma_\mu(V-A\gamma_5)v_s\right]$$

#1

$$= \frac{G_F^2}{2(s-m_Z^2)^2} m_Z^6 \left(\frac{k^2}{m_Z^2} - 4\right) \sum_{s,s'} \{[\bar{v}_s \gamma_\mu (V - A\gamma_5) u_{s'}]$$

$$\left(\frac{k_2^\mu k_2^\nu}{m_Z^2} - g^{\mu\nu}\right) [u_{s'} \gamma_\nu (V - A\gamma_5) v_s]\}$$

$\downarrow$ Casimir's trick

$$= \frac{G_F^2}{2(s-m_Z^2)^2} m_Z^6 \left(\frac{k^2}{m_Z^2} - 4\right) \times \left[Tr[\not{p}_2 \gamma_\mu (V - A\gamma_5) \not{p}_1 \gamma_\nu (V - A\gamma_5)] \left(\frac{k_2^\mu k_2^\nu}{m_Z^2} - g^{\mu\nu}\right)\right]$$

$$Tr[\not{p}_2 \gamma_\mu (V - A\gamma_5) \not{p}_1 \gamma_\nu] = p_2^\lambda p_1^\sigma \, Tr[\gamma_\lambda \gamma_\mu (V - A\gamma_5) \gamma_\sigma \gamma_\nu (V - A\gamma_5)]$$

$$= p_2^\lambda p_1^\sigma \{ V^2 Tr[\gamma_\lambda \gamma_\mu \gamma_\sigma \gamma_\nu] - VA \, Tr[\gamma_\lambda \gamma_\mu \gamma_\sigma \gamma_\nu \gamma_5]$$

$$- VA \, Tr[\gamma_\lambda \gamma_\mu \gamma_5 \gamma_\sigma \gamma_\nu] + A^2 Tr[\gamma_\lambda \gamma_\mu \gamma_5 \gamma_\sigma \gamma_\nu \gamma_5]\}$$

$$= p_2^\lambda p_1^\sigma \{ (V^2 + A^2)(g_{\lambda\mu} g_{\sigma\nu} - g_{\lambda\sigma} g_{\mu\nu} + g_{\lambda\nu} g_{\mu\sigma})$$

$$- 8 i VA \, \epsilon_{\lambda\mu\sigma\nu}\}$$

$$\Rightarrow g^{\mu\nu} \, Tr[\not{p}_2 \gamma_\mu (V - A\gamma_5) \not{p}_1 \gamma_\nu]$$

$$= \{(V^2 + A^2)[(p_2 \cdot p_1) - 4(p_2 \cdot p_1) + (p_2 \cdot p_1)]\}$$

$$= -2(V^2 + A^2)(p_2 \cdot p_1)$$

$$k^\mu k^\nu \, Tr[\not{p}_2 \gamma_\mu (V - A\gamma_5) \not{p}_1 \gamma_\nu] = \{(V^2 + A^2)[2(p_1 k_2)(p_2 k_2) - (p_1 p_2) k_2^2]\}$$

$$\Rightarrow \quad |M|^2 = \frac{G_F}{2(s-m_Z^2)^2} m_Z^6 \left(\frac{k^2}{m_Z^2} - 4\right)(V^2 + A^2) \left[\frac{2(p_1 k_2)(p_2 k_2)}{m_Z^2} - (p_1 p_2) + 3(p_1 p_2)\right]$$

$$= \frac{G_F^2}{2(s-m_Z^2)^2} m_Z^6 \left[\frac{k^2}{m_Z^2} - 4\right] 2(V^2 + A^2) \left\{ (p_1 p_2) + \frac{(p_1 k_2)(p_2 k_2)}{m_Z^2} \right\}$$

(d) Cross Section in COM frame.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_f}{p_i} \times \frac{G_F^2}{2(s-m_Z^2)^2} m_Z^6 \left(\frac{k^2}{m_Z^2} - 4\right) 2(V^2 + A^2) \left\{ (p_1 \cdot p_2) + \frac{(p_1 k_2)(p_2 k_2)}{m_Z^2} \right\}$$

(3) Width of the W boson.

(a) Write all the allowed 2-body decays of the W^- boson in the SM. Neglect quark mixing ($V_{CKM} = 1$)

→ Leptons : $W^- \to e^- \bar{\nu}_e$

$W^- \to \tau^- \bar{\nu}_\tau$

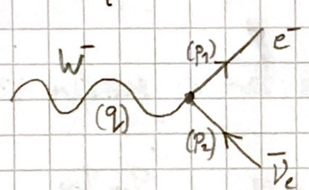
$W^- \to \mu^- \bar{\nu}_\mu$

Quarks :

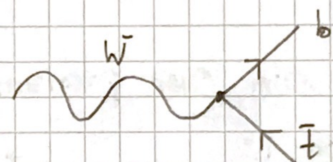
• $W^- \to \bar{u} s$	• $W^- \to \bar{u} b$	• $W^- \to \bar{c} s$
• $W^- \to \bar{u} d$	• $W^- \to \bar{c} d$	• $W^- \to \bar{c} b$
• $W^- \to \bar{t} d$	• $W^- \to \bar{t} s$	• $W^- \to \bar{t} b$

(b) Draw the Feynman diagrams for a decay to Leptons & for a decay to quarks.

→



(Leptons)



(Quarks)

(c) Neglecting quark mixing, the Feynman rule for $W f \bar{f}'$ interaction is the same for quarks & leptons : $i \frac{g}{\sqrt{2}} \gamma_\mu P_L = i \frac{g}{2\sqrt{2}} \gamma_\mu (1-\gamma_5)$

Write down the amplitude for $W^-(q) \to e^-(p_1) \bar{\nu}_e(p_2)$

→ Rules :

- Incoming W-boson : $\epsilon_\mu(q)$
- Electron : $\bar{u}(p_1)$
- Anti-neutrino : $v(p_2)$
- Vertex factor : $-i \frac{g}{\sqrt{2}} \gamma^\mu P_L = i \frac{g}{2\sqrt{2}} \gamma^\mu (1-\gamma_5)$

$$-iM = \epsilon_\mu(q)\, \bar{u}(p_1) . \left(-i \frac{g}{\sqrt{2}} \gamma^\mu (1-\gamma^5)\right) v(p_2)$$

$$M = \frac{g}{\sqrt{2}} \epsilon_\mu(q)\, \bar{u}(p_1) \frac{\gamma^\mu}{2} (1-\gamma^5) v(p_2)$$

If we take weak-charged current as:

$$J^\mu = \bar{u}(p_1)\gamma^\mu \frac{1}{2}(1-\gamma^5)v(p_2)$$

$$M = \frac{g}{\sqrt{2}} \epsilon_\mu(q) J^\mu$$

(d) Take the Fermion masses to be zero, & compute the squared matrix element $\overline{|M|}^2$ for $W^- \to e^- \bar{\nu}_e$ by

→ Averaging over the polarizations of the initial state

→ Summing over spin of final state particles.

Write it in terms of G_F & m_W → $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$, $\frac{m_W}{m_Z} = \cos\theta_W$

& work out the dependence on the particles' momenta, working in W rest frame.

Hint: The sum over polarization states for a massive vector boson is

$$\sum_{pol} \epsilon_\mu(q)^* \epsilon_\alpha(q) = -g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m^2}$$

Also, $\gamma_\mu \gamma^\nu \gamma^\mu = -2\gamma_\nu$

→ ~~Start with Lepton current : $J^\mu = \bar{u}(p_1)\gamma^\mu \frac{1}{2}(1-\gamma^5)v(p_2)$~~

~~• Now use the rest frame of the W^- boson & also $m_W \gg m_e$ to neglect m_{lepton}.~~

~~$q = (m_W, \vec{0})$~~

~~$p_1 = ($~~

~~$|M|^2 = MM^* = \sum \frac{g^2}{2} \epsilon_\mu^{(\lambda)}(q) J^\mu_{(-)} (\epsilon_\alpha^{(\lambda)*})^* (J^\alpha_{(-)})^*$~~

~~$= \frac{g^2}{2} \sum \epsilon_\mu^{(\lambda)} (\epsilon_\alpha^{(\lambda)})^* \sum J^\mu$~~

$$\to |M|^2 = MM^* = \overbrace{\frac{1}{3}\sum_{\lambda}}^{\text{Avg over pol}^n} \overbrace{\sum_{\sigma,\sigma'}}^{\text{Sum over final}} \frac{g^2}{2}\, \epsilon_\mu^{(\lambda)}(q)\, \epsilon_\alpha^{(\lambda)}(q)^*\, J^\mu_{(\sigma),(\sigma')} J^\alpha_{(\sigma),(\sigma')}$$

$$= \frac{g^2}{6}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right] \sum_{\sigma,\sigma'} \bar{u}_{(\sigma)}(p_1)\gamma^\mu \frac{1}{2}(1-\gamma^5)\underbrace{v_{(\sigma')}(p_2)\,\bar{v}_{(\sigma')}(p_2)}_{\sigma' \text{ sum} \Rightarrow \text{Dirac sum}}\frac{\gamma^\alpha}{2}(1-\gamma^5)\, u_{(\sigma)}(p_1)$$

$$= \frac{g^2}{6}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right] \sum_{\sigma} \bar{u}_{(\sigma)}(p_1)\gamma^\mu \frac{1}{2}(1-\gamma^5)(\not{p}_2 + \underset{0}{\underset{\shortparallel}{m_{\nu_e}}})\frac{\gamma^\alpha}{2}(1-\gamma^5)\, u_{(\sigma)}(p_1)$$

Sum over σ
$\Rightarrow$ Dirac sum

$$= \frac{g^2}{6\cdot 4}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right] \gamma^\mu(1-\gamma^5)(\not{p}_1 + \underset{0}{\underset{\shortparallel}{m_{e^-}}})\,\not{p}_2\,\gamma^\alpha(1-\gamma^5)$$

$$= \frac{g^2}{24}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right] \gamma^\mu(1-\gamma^5)\gamma^\beta(p_1)_\beta\, \gamma^\nu(p_2)_\nu\, \gamma^\alpha(1-\gamma^5)$$

$$= \frac{g^2}{24}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right](p_1)_\beta(p_2)_\nu\, \mathrm{Tr}\left[\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\alpha(1-\gamma^5)\right]$$

$$= \text{———} \;\text{"}\; \text{———}\; \mathrm{Tr}\left[\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\alpha - \gamma^\mu\gamma^\beta\gamma^\nu\gamma^\alpha\gamma^5\right]$$

$$= \text{———} \;\text{"}\; \text{———}\left(\mathrm{Tr}\left[\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\alpha\right] - \mathrm{Tr}\left[\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\alpha\gamma^5\right]\right)$$

$$= \text{———} \;\text{"}\; \text{———}\left[4\left(g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\beta\alpha} + g^{\mu\alpha}g^{\beta\nu}\right) \cancel{- 4i\epsilon^{\mu\beta\nu\alpha}}\right]$$

Doesn't contribute.

$$= \frac{g^2}{4}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right](p_1)_\beta(p_2)_\nu\left[g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\beta\alpha} + g^{\mu\alpha}g^{\beta\nu}\right]$$

$$= \frac{g^2}{4}\left[-g_{\mu\alpha} + \frac{q_\mu q_\alpha}{m_W^2}\right]\left((p_1)^\mu(p_2)^\alpha - (p_1)^\alpha(p_2)^\mu + g^{\mu\alpha}(p_1)\cdot(p_2)\right]$$

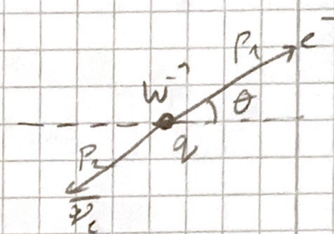
$$= \frac{g^2}{4}\left[\cancel{-(p_1\cdot p_2)} + \cancel{(p_1\cdot p_2)} - 4(p_1\cdot p_2) + \frac{1}{m_W^2}\left[\cancel{(p_1\cdot q)(p_2\cdot q)} - \cancel{(p_1\cdot q)(p_2\cdot q)} + (q\cdot q)(p_1\cdot p_2)\right]\right]$$

$$= \frac{g^2}{4}\left[-4(p_1\cdot p_2) + \frac{(q\cdot q)(p_1\cdot p_2)}{m_W^2}\right]$$

Now take the rest frame for W^- in $W^- \to e^- \bar{\nu}_e$

$$q = (m_W, \vec{0}) \quad , \quad p_1 = (E, E\sin\theta, 0, E\cos\theta)$$
$$p_2 = (E, -E\sin\theta, 0, -E\cos\theta)$$

$$\Rightarrow (p_1 \cdot p_2) = 2E^2 \quad , \quad (q \cdot q) = m_W^2$$



Gmingus

$$|M|^2 = \frac{g^2}{4}\left[-4 \cdot 2E^2 + m_W^2 \frac{2E^2}{m_W^2}\right]$$
$$= -\frac{g^2}{\cancel{4}} \cancel{4}\, 2E^2 + \frac{g^2}{4} \cancel{m_W^2} \frac{2E^2}{\cancel{m_W^2}}$$
$$= -g^2\, 2E^2 + g^2 \frac{E^2}{2}$$
$$= -\frac{3}{2} E^2 g^2$$
$$= -\frac{3}{2} g^2 m_W^2$$

Some mistake, I should get $\frac{1}{3} g_W^2 m_W^2$

- One can also find that

$$|M_-|^2 = g_W^2 m_W^2 \frac{1}{4}(1+\cos\theta)^2$$
$$|M_L|^2 = g^2 m_W^2 \frac{1}{2}\sin^2\theta$$
$$|M_+|^2 = g_W^2 m_W^2 \frac{1}{4}(1-\cos\theta)^2$$

Three possible W polarization modes.

(e) Using this result, ~~compute the total widths of~~ write down $|\overline{M}|^2$ for $W^- \to \bar{u}d$. What are the changes compared to the leptonic one.

~~$\Gamma(W^- \to e^-\bar{\nu}_e)$~~

(Lowest order) → $\Gamma(W^- \to \bar{u}d) = 6\,\Gamma(W^- \to e^-\bar{\nu}_e)$

I am assuming due to the heavier weight of quarks & more generations?

(f) $d\Gamma = \frac{1}{64\pi^2 m} |\bar{M}|^2 d\Omega$ $\qquad d\Omega = d\phi \, d\cos\theta$

- Compare to scattering formula. $(2 \to 2)$

→ Pretty much the same

$$d\sigma = \frac{1}{64\pi^2 s} |M|^2 d\Omega$$

→ We have 2 momenta coming in.

- Compute the partial widths $W^- \to e^- \bar{\nu}_e$
  $W^- \to \bar{u} d$

$W^- \to e^- \bar{\nu}_e$

$$\int d\Gamma = \int \frac{1}{64\pi m_w^2} |\bar{M}|^2 d\Omega = \frac{1}{64\pi \cancel{m_w^2}} \iint \frac{1}{3} g^2 \cancel{m_w^2} \, d\Omega$$

$$\Gamma_{e\nu} = \frac{4\cancel{\pi} g^2}{64 \cancel{\pi}} = \frac{g^2}{16}$$

$W^- \to \bar{u} d$

$$\Gamma_{ud} = 6 \, \Gamma_{e\nu} = \frac{3}{8} g^2$$