MOCK EXAM TP2 - Electrodynamics

Freshmen Tutorial Summer Semester 2019
Syllabus : Griffiths, Chapter 1-3
June 12, 2019

Rohan Kulkarni

Institute of Theoretical Physics, University Leipzig

Full Name		Matriculation no.

Question	1	2	3	4	**Total ↓**
Marks					
Out of	12	8	8	10	40

Information Regarding the Exam

- Exam time : 3:15 - 5:30.
- You will be allowed to use the following objects :
    1. A physical writing instrument. (Pen, Pencil, etc) - Solutions in *ink* only!
    2. Empty sheets of paper. (Blank/Ruled/Chequered/Dotted)
    3. Your own brain.
- Use of all electronic devices is strictly prohibited. Anyone using any digital devices will be barred from the exam.
- No communication during the exam. If you have any questions just raise your hand.
- You may have lots of choices in your life but sadly here, you have none. The exam has 4 questions, all questions are compulsory.
- **Give reasonable explanations to your calculations/answers. The explanation can be one liner or a paragraph, as long as it contains the keywords it will be considered valid.**

You need 50% to pass the exam.[1] Good Luck![2]

Question 1 : Conceptual Questions [12p+2p*]

1. Suppose you have a perfect ball of copper with radius R (by perfect we mean it satisfies spherical symmetry and it is filled). The total charge in/on this sphere is Q. {2p}
    i. Calculate/Write down the Electric field for $r < R$ [1p]
    ii. Calculate/Write down the Electric field for $r > R$ [1p]
2. Derive the Laplace equation using Gauss's law/Maxwell's first equation and the definition of a electric potential. {2p}
3. Maxwell's equation and Vector calculus {2p}
    i. Write down the two Maxwell's equations relevant to electrostatics [1p]
    ii. For both the equations write a statement explaining what they mean physically [1p]
4. Explain the technique of image charges. In a typical image charge problem, what is the physical quantity you are finding? {2p}
5. Write down the formulae for the following {3p}
    i. Electric field at a point p due to n point charges with n'th point having the charge q_n. (Do not assume the point p to be the origin) [1.5p]
    ii. Induced charge density for a conductor due to a potential V or an electric field $\vec{E}$. (σ) [1.5p]

1. Unless the average score falls below 50%. Then the passing percentage will be recalculated accordingly. (So basically, don't give up! You could score really decent assuming the fact that even others find the exam difficult). If you have studied, you won't need the luck.

2. If you have studied, you won't need the luck. You may ignore it then ;).

6. Using Dirac delta functions and step functions in appropriate coordinates, express the following charge distributions as three-dimensional charge densities $\rho(\vec{r})$. {1p+2p*}
   i. A charge Q evenly distrubuted over the surface or a sphere with radius R. [1p]
   ii. In Cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius b. (BONUS) [2p*]

QUESTION 2 : ELECTROSTATICS I [8P]

Suppose you are given a *cylindrically symmetric* charge density $\rho(\vec{r})$ with the total charge Q. Inside the cylinder of radius R and height d there is charge density $\rho(\vec{r})$ and no charge outside. Setup the cylinder in such a way that the length is parallel to z axis and the origin is at the center of the cylinder. Draw a diagram. (You can assume $d \gg R$)

$$\rho(\vec{r}) = \alpha\, r\, \Theta(R-r)\Theta\left(\frac{d}{2}-z\right)\Theta\left(\frac{d}{2}+z\right)$$
$$\text{where} \ : \ r = \sqrt{x^2+y^2}$$

1. Obtain the constant α as a function of R, Q and d (Other than that you can have numerical factors). While doing this computation, show how you take care of the step functions explicitly. [2p]
2. Use Gauss's law to find the Electric field inside this cylinder at a distance s_n from the z-axis. [Hint : Use a Gaussian surface for cylindrical symmetry] [3p]
3. Use Gauss's law to find the Electric field outside this cylinder at a distance s_p from the z-axis. [Hint : Use a Gaussian surface for cylindrical symmetry] [3p]

QUESTION 3: ELECTROSTATICS II [8P]

Consider a conducting sphere with radius R and the potential on its surface specified by

$$V_0 = \alpha\,(\cos\theta + 3) + \beta \tag{2.1}$$

where α, β are constants.

1. Write down the reason/s why can we use the following the ansatz [1p]

$$V(r,\theta) = \sum_{l=0}^{l=\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$
$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = (3x^2-1)/2$$

   to find our potential.
2. Write down all the appropriate boundary conditions. [2p]
3. Find the potential outside the conductor. [3p]
4. Find the electric field inside the conductor. [2p]

QUESTION 4 : ELECTROSTATICS III [10P]

Two infinitely-long grounded metal plates at $y=0$ and $y=a$ are connected at $x=\pm b$ by metal strips maintained at a constant potential V_0. Our goal is to find the potential inside the resulting hollow rectangular pipe.

1. Draw a diagram for this situation. [2p]
2. What technique/equation will you use in order to reach our goal? Why? [1p]
3. Write down the Boundary appropriate boundary conditions for the potential. (4 in total) [2p]
4. Derive the necesary equation

$$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin(ky) + D\sin(ky)) \tag{1}$$

   for the potential. (Hint : Look at point 2 and solve the equations) [2p]
5. Impose the boundary conditions from (3) and use the equation (4) to solve them. (Don't forget to construct the general linear combination, we should have a sum in the solution). [3p]

——————————*——————————